5. SYSTEM OF RADIATORS

Modern aerials are complex systems of radiators. It is caused by very inconsistent requirements to aerials. Their compliance with the help of aerials with a continuous distribution of field sources encounters with significant difficulties, and in some cases it is impossible. Use of the radiator system enables in many cases to simplify decisions of such problems as formation DD of special form, control of DD form, control of DD orientation in space, scanning in space, increase of radiation power, etc. Elements of antennas systems are radiators with a continuous distribution of field sources. The last may be considered as a system of elementary radiators. Thus, in the aerial system the radiating elements may be distributed discretely or continuously. When the antenna system consists of discretely placed single-type radiators, it is named the aerial array. Ample opportunities of antennas arrays promote their wide application in radio engineering.

5.1. The pattern multiplication theorem

A group of radiators, discretely located on certain distances one from another (Fig. 5.1), forms an antenna array. It is known, that in the antenna arrays radiators with identical directed properties are used.

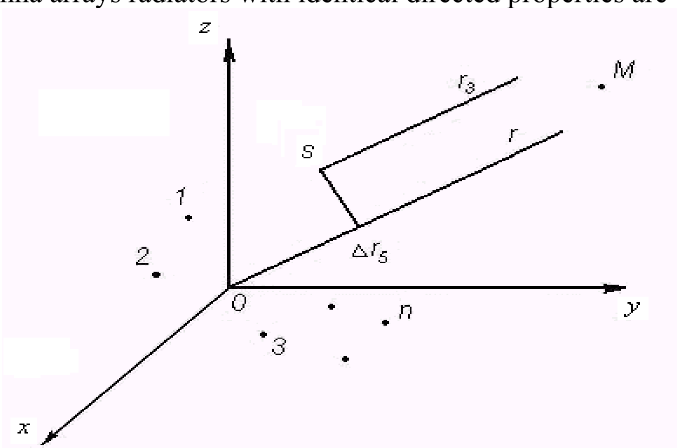


Fig. 5.1

Let us find a radiation field of the considered system. The field intensity created by radiator S is defined as

$$\dot{E}_S = E_{max,S} F_S(\theta,\varphi) e^{-i(kr_S - \psi_S)}, \qquad (5.1)$$

where $E_{max,S}$ is the amplitude multiplier, which value depends on the excitation intensity of radiator S, its construction and distance from the radiator to the point of observation; $F_S(\theta,\varphi)$ is DC of radiator S; r_S is the distance from radiator S to the point of observation; ψ_S is the phase of electromagnetic oscillations, which feed radiator S.

It is obvious, that the field intensity of radiators with identical space orientation is equal to the sum of the field intensity of separate radiators:

$$\dot{E} = \sum_{S=1}^{n} \dot{E}_S . \qquad (5.2)$$

We can assume, that for identical radiators

$$F_1(\theta,\varphi) = F_2(\theta,\varphi) = \ldots = F_s(\theta,\varphi) = \ldots = F_n(\theta,\varphi).$$

Then expression (5.2) after substitution of the intensity value (5.1) is

$$\dot{E} = E_{max,1} F_1(\theta,\varphi) \sum_{S=1}^{n} a_S e^{-i(kr_S - \psi_S)}, \qquad (5.3)$$

where $a_S = E_{max,s} / E_{max,1}$ is the relative amplitude of electromagnetic oscillations, which feed the radiator S.

Let us designate a distance from the origin of coordinates to the point of supervision through r, a path-length difference of the radiator located in the origin of coordinates and radiator S through Δr_S. Then

$$r_S = r - \Delta r_S .$$

Let us take the common multiplier outside the sum of expression (5.3)

$$\dot{E} = E_{max,1} F_1(\theta,\varphi) e^{-ikr} \sum_{S=1}^{n} a_S e^{i(k\Delta r_S + \psi_S)}. \qquad (5.4)$$

In this formula the sum of harmonious members can be designated as

$$\sum_{S=1}^{n} a_S e^{i(k\Delta r_S + \psi_S)} = \tilde{f}_{\Sigma}(\theta,\varphi) = f_{\Sigma}(\theta,\varphi)e^{i\psi(\theta,\varphi)},$$

(5.5)

that enables to result formula (5.4) in the following form

$$\dot{E} = E_{max,1}F_1(\theta,\varphi)f_{\Sigma}(\theta,\varphi)e^{-ikr}e^{i\psi(\theta,\varphi)}.$$

Hence, it follows that DC of the array is the product of two multipliers:

$$f(\theta,\varphi) = F_1(\theta,\varphi)f_{\Sigma}(\theta,\varphi), \qquad (5.6)$$

where the first one describes directional properties of the single radiator, and the second multiplier - the array factor (AF) takes into account the result of summation of the field intensity of the whole system. The array factor is sometimes called the pattern factor or the space factor. In expression (5.5) function $\psi(\theta,\varphi)$ is the phase directional characteristic.

The physical meaning of the array factor can be determined from expression (5.6), if to agree, that elements of an array are non-directional radiators, for which the DC value does not depend on coordinate angles, that is $F(\theta,\varphi) = 1$. At this condition, the DC of the array, as seen from expression (5.6), is equal to:

$$f(\theta,\varphi) = f_{\Sigma}(\theta,\varphi).$$

So, the array factor takes into account an interference of waves, which are radiated by separate elements of the array. In the case of the array, formed by n non-directional radiators, the array factor is the DC of the whole aerial.

Thus, the DC of the system, consisting of identical and equally directed radiators, is the product of the separate radiator's DC and the DC of the same system, but consisting of non-directional radiators.

5.2. Radiation impedance of coupled antennas

A radiator, being an element of an aerial system, is in an electromagnetic field established by other elements of the system. Therefore in the considered radiator currents will be induced under the action of this electromagnetic field. Because of these currents the operation mode of the radiator will be defined not only by its own parameters, but also by the character and level of the influence of nearby radiators.

To estimate this influence on the radiation impedance let us consider the aerial system as an electric circuit, consisting of n elements. Assuming, that the array is situated in the linear space, we may to write down

$$\begin{aligned}
\dot{U}_1 &= Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 + \ldots + Z_{1s}\dot{I}_s + \ldots + Z_{1n}\dot{I}_n;\\
\dot{U}_2 &= Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 + \ldots + Z_{2s}\dot{I}_s + \ldots + Z_{2n}\dot{I}_n;\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
\dot{U}_s &= Z_{s1}\dot{I}_1 + Z_{s2}\dot{I}_2 + \ldots + Z_{ss}\dot{I}_s + \ldots + Z_{sn}\dot{I}_n;\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
\dot{U}_n &= Z_{n1}\dot{I}_1 + Z_{n2}\dot{I}_2 + \ldots + Z_{ns}\dot{I}_s + \ldots + Z_{nn}\dot{I}_n,
\end{aligned} \qquad (5.7)$$

where $\dot{U}_s$ is the voltage across terminals of radiator s ; $\dot{I}_s$ is the current, flowing through terminals of radiator s ; Z_{ss} is the internal impedance of radiator s ; Z_{sq} is the mutual impedance between s and q radiators; $s = 1,2,3,\ldots n$; $q = 1,2,3,\ldots n$. The passive linear electric circuit follows the reciprocity principle, therefore we can assume, that $Z_{sq} = Z_{qs}$. It is obvious, that total impedance Z_s of radiator s is determined as

$$Z_s = \frac{\dot{U}_s}{\dot{I}_s} = \sum_{q=1}^{n} \frac{\dot{I}_q}{\dot{I}_s} Z_{sq} .$$

So, the total impedance of an element of the aerial system represents the sum of the internal impedance and induced impedances:

$$Z_s = Z_{ss} + \sum_{q=1, q \neq s}^{n} \frac{\dot{I}_q}{\dot{I}_s} Z_{sq} .$$

The impedance in radiator s induced by the electromagnetic field of radiator q is determined by the mutual impedance and the ratio of currents, which feed these two radiators:

$$Z_{in,sq} = \left(\frac{\dot{I}_q}{\dot{I}_S} \right) Z_{sq}, \qquad (5.8)$$

or

$$Z_{in,sq} = m_{sq} Z_{sq} e^{i\psi_{Sq}},$$

where m_{sq} is the ratio of currents' amplitudes of q and s radiators; ψ_{sq} is the phase shift between currents of q and s radiators.

The mutual impedance of two dipoles can be found by the method of induced EMF. Let us consider a system, which consists of two antennas (Fig. 5.2). The distribution of currents along dipoles is known. For the first radiator the current on distance z from terminals is

$$\dot{I}_1(z) = \dot{I}_1 \tilde{f}_1(z), \qquad (5.9)$$

where $\tilde{f}_1(z)$ is the function of the current distribution along the first dipole (generally it may be complex).

At excitation of the second radiator the electric field with intensity $\dot{E}_{12}$ appears near the surface of the first radiator. The tangent component of this field intensity on a surface of the first radiator is equal to $\dot{E}_{z12}$. But on a conductor surface according to the boundary conditions the tangent component of a vector E should have zero value. To satisfy the boundary conditions the additional field due to energy of the power supply of the first aerial must appear. The tangent component of this field $\dot{E}'_z$, should satisfy to the equation

$$\dot{E}'_z = - \dot{E}'_{z12}.$$

Thus, in an aerial element dz, the EMF will be

$$d\dot{\mathcal{E}} = - \dot{E}'_{z12} dz$$

As the value of the current in the element dz (5.9) is known, the complex power, which is spent by the power supply to create the field with intensity $\dot{E}'_z$ near the element dz, is:

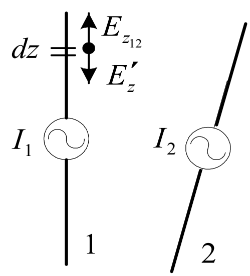


Fig. 5.2

$$d\widetilde{P}_{12} = \frac{1}{2} d\dot{\mathcal{E}}\dot{I}_1^*(z) = -\frac{1}{2}\dot{E}_{z12}\dot{I}_1^*(z)dz$$
.

The total additional power is defined as:

$$\widetilde{P}_{12} = -\frac{1}{2}\int_{-l}^{l}\dot{E}_{z12}\dot{I}_1^*(z)dz\ ,$$

where l is the arm length of the dipole.

The additional power, spent by the power supply, which is connected to terminals of the first dipole, can be considered as the power, which is caused by the induced impedance. Therefore

$$Z_{in12} = \frac{2\widetilde{P}_{12}}{\dot{I}_1\dot{I}_1^*} = -\frac{1}{\dot{I}_1\dot{I}_1^*}\int_{-l}^{l}\dot{E}_{z12}\dot{I}_1^*(z)dz\ . \qquad (5.10)$$

Taking into account, that the field intensity is proportional to the current amplitude of the second radiator

$$\dot{E}_{z12} = \widetilde{e}_{12}(z)\dot{I}_2\ ,$$

where $\widetilde{e}_{12}(z)$ is the factor of proportionality, which depends on a relative positioning of antennas and coordinate z .

Taking into account the latter equation and formula (5.9), expression (5.10) is written down as

$$Z_{in12} = \frac{\dot{I}_2}{\dot{I}_1}\left\{-\int_{-l}^{l}\widetilde{e}_{12}(z)\widetilde{f}_1(z)dz\right\}. \qquad (5.11)$$

Comparing expressions (5.8) and (5.11), we receive the formula for calculation of the mutual impedance:

$$\dot{Z}_{12} = -\int_{-l}^{l}\widetilde{e}_{12}(z)\widetilde{f}_1(z)dz\ .$$

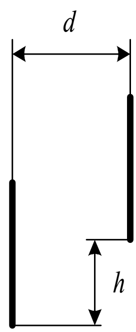


Fig. 5.3

Mutual impedances for dipoles, calculated from the received formula, are given in many monographs on aerials. The best known case is two half-wave dipoles placed in parallel to each other on distance d and shifted along dipoles axes on value h (Fig. 5.3).

The dependence of the active component of the mutual impedance of half-wave dipoles at $h = 0$ on the relative distance between antennas d/λ is represented in Fig. 5.4, and the reactive component - in Fig. 5.5. Oscillatory character of curves is explained by change of the field intensity phase E_{Z12} at alteration of distance d, owing to which the phase shift between current $\dot{I}_1(z)$ in the dipole and EMF $d\mathcal{E}$, induced by EMF, changes.

The damping character of curves is explained by reduction of intensity E_{Z12}, when distance between dipoles increases.

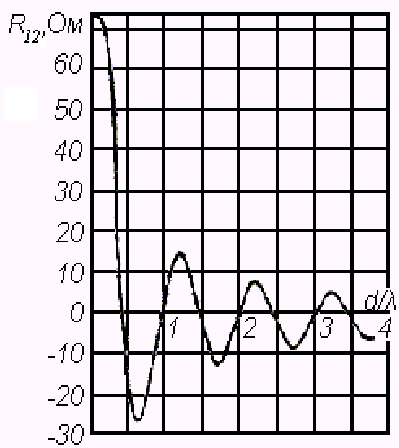


Fig. 5.4

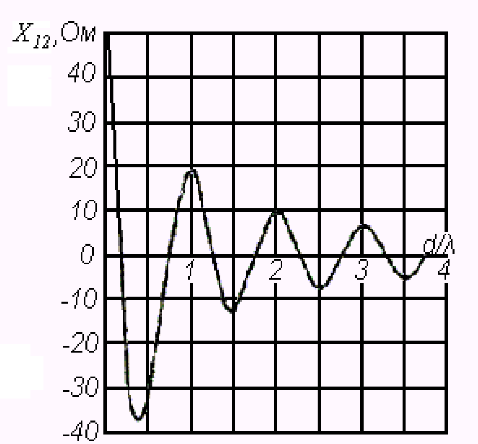


Fig. 5.5

5.3. Linear array

Arrays discretely located along a straight line are often used in the aerial engineering. Let distances between two adjacent array elements be identical and equal to d (Fig. 5.6). It is the so-called equally spaced linear array. All radiators are numbered from one to n. Besides, the origin of the spherical coordinate system is placed in the middle of the first radiator, and the polar axis (axis z) coincides with the lining up of antennas.

Let us assume, that all array elements are fed by electromagnetic oscillations of an equal amplitude but each element has a phase angle, which lags behind by a constant with respect to the phase angle of an adjacent radiator of the lower ordinal number. Therefore, in formula (5.4) it is necessary to agree, that

$$a_1 = a_2 = a_3 = \ldots = a_s = \ldots = a_n = 1$$

and

$$\psi_1 = 0; \psi_2 = -\psi; \psi_3 = -2\psi; \ldots; \psi_s = -(s-1)\psi; \ldots; \psi_n = -(n-$$

The path-length difference from the first and s -th array element up to the observation point, as apparent from Fig. 5.6, is equal to:

$$\Delta r_S = (s-1) d\, cos\, \theta \, ,$$

where θ is the angle between the straight line, on which radiators are placed, and the direction to the observation point.

Taking into account given remarks, expression (5.4) is written down as

$$\dot{E} = E_{max,1} F_1(\theta, \varphi) e^{-ikr_1} \sum_{s=1}^{n} e^{i(s-1)(kd\, cos\theta - \psi)} \, .$$

(5.12)

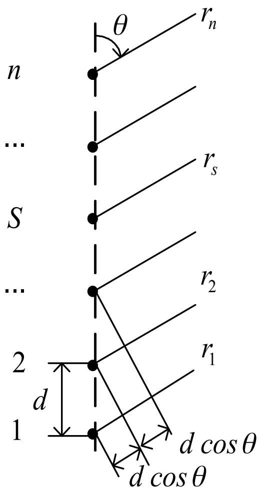


Fig. 5.6

The array factor in the right part of equation (5.12) is the sum of n member of geometrical progression, which first member is equal to one, and the denominator is

$$q = e^{i(kd\, cos\,\theta - \psi)} \, .$$

Using the known formula of summation

$$S_n = \frac{1 - q^n}{1 - q} \, ,$$

we find, that

$$\dot{E} = E_{max,1} F_1(\theta, \varphi) e^{-ikr_1} \frac{1 - e^{in(kd\, cos\theta - \psi)}}{1 - e^{i(kd\, cos\theta - \psi)}} \, . \qquad (5.13)$$

If from this expression in numerator to take out multiplier $e^{i\frac{n}{2}(kd\, cos\,\theta - \psi)}$, and in denominator $e^{i\frac{1}{2}(kd\, cos\,\theta - \psi)}$, then the formula for the field intensity will take the following form

$$\dot{E} = E_{max,1} F_1(\theta,\varphi) \frac{\sin\left[\frac{n}{2}(kd\cos\theta - \psi)\right]}{\sin\left[\frac{1}{2}(kd\cos\theta - \psi)\right]} e^{i\psi_0} e^{ikr}. \qquad (5.14)$$

(5.14)

Where $\psi_0 = 0.5(n-1)\psi$ is the phase of oscillations, which feed the aerial in the centre of the linear array, $r = r_1 - 0.5(n-1)d\cos\theta$ is the distance from the array centre to the point of observation.

The AF of the linear array, as seen from formula (5.14)

$$f_\Sigma(\theta) = \frac{\sin\left[\frac{n}{2}(kd\cos\theta - \psi)\right]}{\sin\left[\frac{1}{2}(kd\cos\theta - \psi)\right]}. \qquad (5.15)$$

Under condition

$$kd\cos\theta = \psi \qquad (5.16)$$

the array factor reaches the maximal value. Using (5.16) we can show, that the AF maximal value is equal to n, therefore, the normalized value of the array factor is

$$F_\Sigma(\theta) = \frac{\sin\left[\frac{n}{2}(kd\cos\theta - \psi)\right]}{n\sin\left[\frac{1}{2}(kd\cos\theta - \psi)\right]}. \qquad (5.17)$$

To simplify the AF analysis let us introduce the generalized angular coordinate

$$F_\Sigma(u) = \frac{\sin\frac{nu}{2}}{n\sin\frac{u}{2}}. \qquad (5.18)$$

The graphic dependence of the $F_\Sigma(u)$ absolute value on coordinate u is represented in Fig. 5.7.

As shown in the figure, at

$$u = 2p\pi, \qquad (5.19)$$

where $p = 0, \pm 1, \pm 2, \pm 3$, function $|F_\Sigma(u)|$ reaches maximum that corresponds to the most intensive radiation of the array. The value of the generalized angular coordinate (5.19) determines the direction of the maximal radiation.

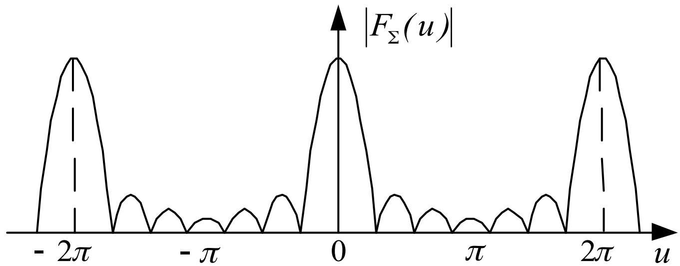


Fig. 5.7

Function $|F_\Sigma(u)|$ takes zero values if the numerator of expression (5.18) becomes equal to zero at the denominator differs from zero. Hence, it follows that the generalized angular coordinate, which defines directions of the zero radiation, should be equal

$$u = \frac{2p\pi}{n} \quad at \quad p = \pm 1, \pm 2, \pm 3, \ldots, \text{ and } p \neq nm, \quad (5.20)$$

where m is integer number.

The diagram, presented in Fig. 5.7, can be considered as the generalized DC. Therefore, besides major lobes, which direction is determined by equation (5.19), there are lobes of smaller intensity (side lobes). In order to define the direction of the maximal radiation of side lobes, it is necessary to take into account, that at big values n (large number of array elements) the argument of the sine wave function in the numerator of formula (5.18) varies much faster in comparison with the argument of the sine wave function in the denominator. Therefore, values of side lobes maximums can be determined from an approximate equation

$$sin\frac{nu}{2} = \pm 1.$$

Hence, generalized angular arguments for directions of side lobes maximums are

$$u = \frac{2p+1}{n}\pi, \qquad (5.21)$$

where $p = +1, \pm 2, \pm 3, \ldots$

Number p can not take the value equal to zero. As seen from Fig. 5.7, the argument will be between the main maximum ($u = 0$) and the first zero ($u = 2\pi/n$). It cannot take value $p = -1$ as well as the argument is in the interval from the first zero $(u = -2\pi/n)$ to the main maximum $(u = 0)$.

Using expressions (5.19) - (5.21), let us find directions of the DD major lobes, zero and side lobes for the linear array, which consists of non-directional radiators:

$$cos\,\theta_{max,p} = \frac{2p\pi}{kd} + \frac{\psi}{kd}; \quad p = 0, \pm 1, \pm 2, \pm 3, \ldots; \qquad (5.22)$$

(5.22)

$$cos\,\theta_{0p} = \frac{2p\pi}{nkd} + \frac{\psi}{kd}; \quad p = \pm 1, \pm 2, \pm 3, \ldots, p \neq nm; \qquad (5.23)$$

$$\cos\theta_{mp} = \frac{(2p+1)\pi}{nkd} + \frac{\psi}{kd}; \quad p = +1, \pm 2, \pm 3, \ldots,$$

where $\theta_{max,p}$ is the direction of the main maximum of the order p; θ_{0p} is the direction of zero radiation of the order p; θ_{mp} is the direction of the maximum of the side lobe of the order p.

Taking into account, that $|cos\,\theta| \leq 1$ we can determine the number of the DD major lobes, the side lobes and the zeros for the linear array by formulas (5.22) and (5.23).

5.4. Radiation of cophased array

In a cophased aerial array, all radiators are fed in the same phase. Therefore it is necessary to agree, that $\psi = 0$. Thus, the array factor (5.17) becomes simpler:

$$F_{\Sigma}(\theta)=\frac{\sin\left(\dfrac{nkd\cos\theta}{2}\right)}{n\sin\left(\dfrac{kd\cos\theta}{2}\right)}. \qquad (5.24)$$

It follows from formula (5.22), that at $kd<2\pi$ the array DD will contain one major lobe. By the same formula, the direction of the main maximum can be determined as:

$$\cos\theta_{max}=0.$$

So, the maximal radiation direction of the cophasal linear array coincides with the perpendicular to the line of the radiator arrangement.

Transition from discrete to continuous distribution of field sources is carried out under condition, that the distance between adjacent elements decreases up to zero $(d\to 0)$, and the quantity of elements grows indefinitely $(n\to\infty)$. Making the limiting transition in (5.24)

$$\lim_{\substack{n\to\infty\\ d\to 0}} F_{\Sigma}(\theta)=\lim_{\substack{n\to\infty\\ d\to 0}}\frac{\sin\left(\dfrac{nkd\cos\theta}{2}\right)}{n\sin\left(\dfrac{kd\cos\theta}{2}\right)}.$$

and taking into account, that the sine of the small argument with the big accuracy coincides with the argument, and product $nd=L$ is the length of the considered aerial system, we receive

$$F_{\Sigma}(\theta)=\frac{\sin\left(\dfrac{kL\cos\theta}{2}\right)}{\dfrac{kL\cos\theta}{2}}.$$

Very frequently angle θ is reckoned not from the aerial axis, but from the perpendicular to the axis (from the direction of the maximal radiation). Then

$$F_{\Sigma}(\theta)=\frac{\sin\left(\dfrac{kL\sin\theta}{2}\right)}{\dfrac{kL\sin\theta}{2}}.$$

Let us consider features of the array factor of the linear aerial system. The direction of the zero radiation in the array DD according to formula (5.23) is

$$cos\,\theta_{op} = \frac{p\lambda}{nd};\quad p = \pm1, \pm2, \pm3, \ldots, p \neq nm.$$

It is obvious, that directions of the zero radiation, which limit the major lobe, can be bound at $p = \pm1$:

$$cos\,\theta_{01} = \pm\frac{\lambda}{nd}.$$

If to count angle θ_0 from the maximal radiation direction, then $\theta_0 = 90° - \theta_{01}$ and $sin\,\theta_0 = \lambda/nd$. For arrays of the big length, if $nd >> \lambda$, it is possible to assume, that the beam width on the zero level is equal to $2\theta_0 = 2\lambda/nd$, or $2\theta_0 \approx 115° \dfrac{\lambda}{nd}$.

Under the same condition, the beamwidth on the half power level is

$$2\theta_{0.5} \approx 51° \frac{\lambda}{nd}. \qquad (5.25)$$

The directional diagram of the cophased system, which consists of four nondirective radiators, located on distance $d = \lambda/2$ one from another, is presented in Fig. 5.8. The diagram displays a section of the spatial DD by the plane, which has been carried out through the line of the nondirective radiator arrangement. So, the spatial DD is a surface of rotation of the diagram around the axis $\theta = 0$.

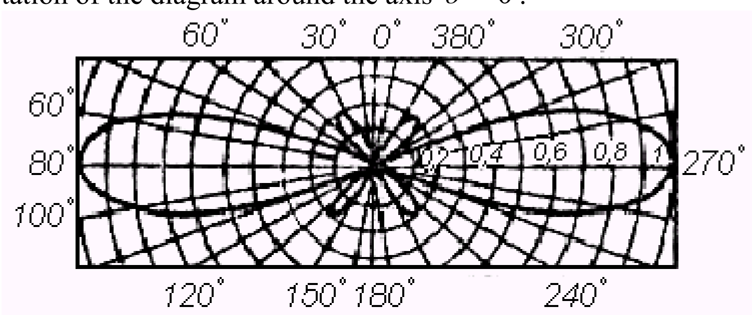


Fig. 5.8

5.5. Linear array of nondirective radiators excited with phase shift

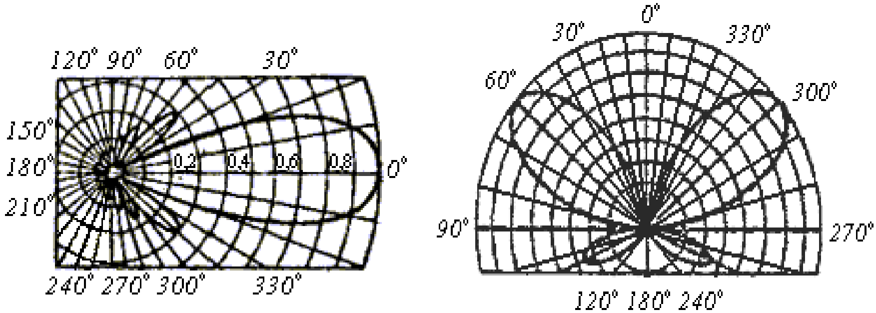
The directional characteristic of a linear array, which comprises nondirective radiators located along a straight line and excited with phase shift, is expressed by formula (5.16).

Directions of the maximal array radiation are determined from expression (5.22). If the distance between adjacent radiators is less than the half of the wavelength $(d < \lambda/2)$, then DD has only one major lobe at $0 \le \theta_{max} \le 180^\circ$. The direction of the maximal radiation at $p = 0$

$$cos\,\theta_{max} = \frac{\psi}{kd}.$$

Changing the phase shift ψ, it is possible to move the direction of the maximal radiation in space. At $\psi = 0$ the direction of the maximal radiation is perpendicular to the line of the radiator arrangement (Fig. 5.8). At $\psi = kd$ the direction of the maximal radiation coincides with the line of radiator arrangement. In this case, waves from separate field sources, which are radiated along the axis of the aerial system, are summarized, that is caused by indemnification of the phase shifts on the feed and phase shifts, which arise at propagation of waves in space. In general, waves from separate radiators, as it was noted earlier, come to the observation point with spatial phases shifts, which is defined by a radiators arrangement and a direction towards the observation point. In a direction $\theta = 0$ (along the line of the radiator arrangement), the spatial phase shift for electromagnetic waves from array elements will be equal to kd. If the phase shift of oscillations, which feed adjacent radiators, is established as $\psi = kd$, then the full indemnification of phase shifts (spatial and on the feed) will be achieved. Therefore the intensity of the field in this direction will be maximal. Choosing distance between array elements $d = \lambda/4$, the phase shift (spatial or on feed currents) equal to 90° can be received. Thus, in the opposite direction these phase shifts are summarized and become equal to 180°, that causes addition of intensity vectors of fields, radiated by adjacent elements, with opposite signs. Therefore at even quantity of array elements the radiation in the opposite direction decreases up to 0. Generally at $d \le \lambda/4$ the radiation level in the opposite direction will be essentially reduced. Thus, the radiation of the array is directed to one side, and it is named the system of the axial radiation.

At $d = \lambda/2$ and $\psi = kd$ the spatial phase shift and the phase shift of oscillations, which feed the radiators, are compensated along a system axis in the forward direction $(\theta = 0)$ as well in the opposite direction $(\theta = 180°)$. Such aerial system will be bi-directional.



a *b*

Fig. 5.9

The array DD at $nd = 4\lambda,\ d = \lambda/4$ and $\psi = kd$ is presented in Fig. 5.9*(a)*. At other values of the phase shift $(\psi < kd)$ the direction of the maximal radiation will occupy some intermediate position. So, for example, in Fig. 5.9*(b)* the DD at $nd = 2\lambda,\ d = \lambda/4$ and $\psi = kd/2$ is shown. This property of the aerial array is used for beam scanning with the purpose of the space coverage in radar and some radionavigating systems.

The array factor (5.16) at limiting transition coincides with the array factor at continuous distribution of field sources:

$$\lim_{\substack{n\to\infty \\ d\to 0}} F_\Sigma(\theta) = \lim_{\substack{n\to\infty \\ d\to 0}} \frac{\sin\left[\frac{nkd}{2}\left(\cos\theta - \frac{\psi}{kd}\right)\right]}{n\sin\left[\frac{nkd}{2}\left(\cos\theta - \frac{\psi}{kd}\right)\right]} = \lim_{d\to 0} \frac{\sin\left[\frac{kL}{2}\left(\cos\theta - \right.\right.}{\frac{kL}{2}\left(\cos\theta - \right.}$$

where ψ/d is the phase shift of oscillations per unit length of the system, $L = nd$ is the length of the antenna system. Physically, ratio ψ/d represents the phase factor β of the electromagnetic wave, extending with some velocity υ_{ph} along the aerial system. Taking it into account, we may enter the following designation:

$$\frac{\psi}{kd} = \frac{\beta}{k} = \frac{2\pi}{\Lambda}\frac{\lambda}{2\pi} = \frac{\lambda}{\Lambda} = \frac{c}{\upsilon} = \xi ,$$

where Λ is the length of wave, propagating along the aerial; ξ is the wave slowing factor. Thus, the array factor with continuous distribution of sources takes the following form

$$f_{\Sigma}(\theta) = \frac{sin\left[\frac{kL}{2}(\xi - cos\,\theta)\right]}{\frac{kL}{2}(\xi - cos\,\theta)} . \qquad (5.26)$$

The direction of maximal radiation

$$\cos\theta_{\max} = \xi . \qquad (5.27)$$

From expression (5.23) zero directions in DD can be found:

$$cos\,\theta_{0p} = \xi - \frac{p\lambda}{nd} \quad at \;\; p = 1,2,3,\dots . \qquad (5.28)$$

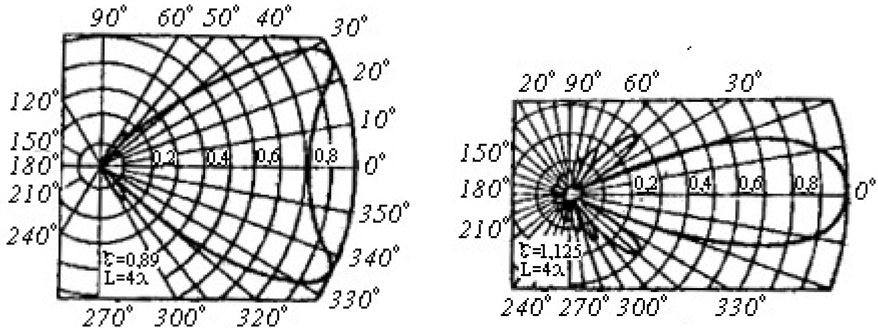
In Fig. 5.9, 5.10, the linear array DDs at wave slowing factors are shown: in Fig. 5.*9(a)* - at $\xi = 1$, on Fig. 5.*9(b)* - at $\xi = 0.5$, in Fig.5.10*(a)* - at $\xi = 0.89$ and in Fig. 5.10, at $\xi = 1.125$. The beamwidth can be determined from formula (5.28) if to agree, that:

$$cos\,\theta_{01} = \xi - \frac{\lambda}{nd} . \qquad (5.29)$$

Hence, provided that $nd >> \lambda$, by decomposing the left part of equation (5.29) in series we obtain

$$2\theta_0 \approx 2\sqrt{\frac{2\lambda}{nd} - 2(\xi - 1)}. \qquad (5.29,a)$$

From given DDs and formulas (5.29, a) it is seen, that the beamwidth decreases at increase of factor ξ. The level of side lobes, thus, is increased as the value of expression maximum (5.26) falls.



a *b*

Fig. 5.10

At $\xi = 1$ and rather big length of the linear array, the beamwidth from expression (5.29, a)

$$2\theta_0 = 2\sqrt{2\frac{\lambda}{nd}}$$

or

$$2\theta_0 = 115^\circ\sqrt{2\frac{\lambda}{nd}}\ .$$

The half-power beamwidth

$$2\theta_0 = 108^\circ\sqrt{\frac{\lambda}{nd}}\ .$$

The directivity factor of the axial radiation system can be determined from formula (2.25), if to use DC (5.26). But it is necessary to take into account, that DC (5.26) for $\xi > 1$ is not normalized. Therefore, the expression for DF at any value of the wave slowing factor is complex. If $\xi = 1$, then from formulas (2.25) and (5.26) it is obtained

$$D_0 = \frac{kL}{\mathrm{Si}2kL - \dfrac{1 - \cos 2kL}{2kL}}\ . \qquad (5.30)$$

For cases, frequently met in practice when $L \geq \lambda$, formula (5.30) is far simpler:

$$D_0 = 4\frac{L}{\lambda}\ . \qquad (5.31)$$

The directivity factor of the axial radiation system at any value of the wave slowing factor is determined from the diagram, represented in Fig. 5.11. The diagram is constructed by the formula for DF, based on the assumption, that $\xi \neq 1$.

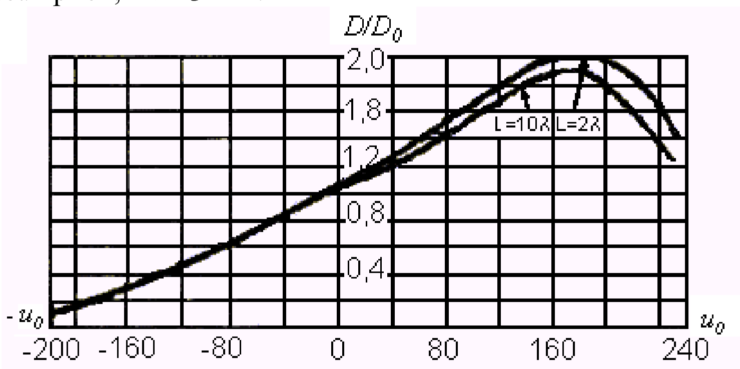


Fig. 5.11

On the horizontal axis of the diagram the difference of the phase progression of the wave in the system and in free space is plotted: $u_0 = kL(\xi - 1)$.

The directivity factor of the axial radiation system diminishes as the wave slowing factor $(\xi < 1, v_{ph} > c)$ reduces. The left part of Fig. 5.11 illustrates this condition. At increase ξ ($u_0 > 0$ in Fig. 5.11), the DF first increases, reaches its maximum, and then again decreases. It is apparent from the diagram, that the DF reaches its maximum at $u_0 = \pi$. From here it is possible to determine the optimum wave slowing factor

$$\xi_{opt} = 1 + \frac{\lambda}{2L},$$

or the optimum length of the axial radiation array:

$$L_{opt} = \frac{\lambda}{2(\xi - 1)}.$$

Change of the DF value is caused by the dependence of the directional properties of the axial radiation antenna on the wave slowing factor. Let us enter, similarly to expression (5.18), the generalized angular coordinate

$$u = \frac{kL}{2}(\xi - \cos\theta).$$

Then the DC (5.26) takes the following form

$$f_\Sigma(u) = \frac{\sin u}{u}. \qquad (5.32)$$

The coordinate u varies in limits from $u_m = 0.5kL(\xi - 1)$ up to $u_{max} = 0.5kL(\xi + 1)$. At $\xi < 1$, the minimum value of coordinate u is less than zero $(u_m = u_{m1})$. At $\xi = 1$ $u_m = 0$. So, at $\xi \geq 1$ the generalized angular coordinate accepts only positive values. The diagram of function (5.32) is presented in Fig. 5.12. Taking into account, that at $\theta = 0$ the coordinate $u = u_m$, it is necessary to accept direction $u = u_m$ as an axis of symmetry, when constructing a generalized DD (in space or in a plane).

It follows from Fig. 5.12, that in the generalized DD in the plane of the given diagram at $\xi < 1$ the major lobe will be wider, than in DD at

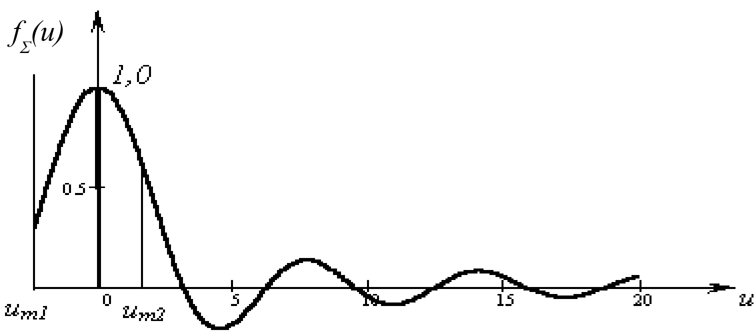


Fig. 5.12

$\xi = 1$. Besides, in the major lobe two directions of the maximal radiation at $\cos\theta = \xi$ will exist. DD presented in Fig. 5.*10(a)* corresponds to this case. At $\xi > 1$, the axis of symmetry of the generalized DD is shifted to position $u = u_{m2}$. In this connection the maximal value of $f_\Sigma(u)$ decreases, and the major lobe becomes narrower. Thus the level of side lobes grows as their intensity has remained constant. This case is presented in Fig. 5.*10(b)*.

The narrowing of the major lobe at displacement of coordinate u_{m2} to the right from zero first is faster, than the reduction of the major lobe level. Therefore, at first, DF increases at increase ξ, too. Then the growth of the wave slowing factor results in faster fall of the maximal value of the major lobe. This is due to the fact that coordinate u_{m2} gets on an abrupt slope of the major lobe of diagram (5.32) and consequently small displacement of coordinate u_{m2}, which makes the major lobe only a little bit narrower, will cause landslide reduction of the function $f_{\Sigma}(u_{m2})$ value - a maximum of the DD major lobe. The level of side lobes, as it is characterized by the ratio of intensity in maximum of side lobes to the major lobe maximum, also increases simultaneously. In this connection the growth of the DF slows down and after achieving some maximum DF decreases (Fig. 5.11). As the dependence of u_{m2} on the wave slowing factor is almost the same, as on the axial radiation system length, dependence DF on the length of the aerial in right or left quadrant of Fig. 5.11 has the same character, as the DF dependence on the wave slowing factor (under condition that $\xi>1$).

5.6. Wire with running wave of current

The linear array of continuously distributed radiators, which are excited with the phase shift, is realized as a wire, in which running wave of current propagates. For maintenance of the running wave mode the wire is loaded on resistance, which value equals the wave resistance of the wire. Let us assume, that the wire is situated on the big distance from the ground and other bodies, due to what influence of any objects on the radiation field is excluded.

Let us agree, that the current amplitude does not change along all length of the wire, and the phase changes linearly

$$\dot{I}_z = I_A e^{-i\beta z}, \qquad (5.33)$$

where I_A is the current in the beginning of the wire; β is the phase factor of the wave in the wire.

The propagation velocity of the wave along the wire may differ from the light velocity, therefore

$$\beta = \frac{2\pi}{\Lambda} = \frac{2\pi}{\lambda}\frac{\lambda}{\Lambda} = k\frac{c}{\upsilon_{ph}} = k\xi, \qquad (5.34)$$

where Λ is the wave length in the wire; υ_{ph} is the phase velocity of propagation in the wire; c is the light velocity.

Let us consider the wire as a number of elementary sources (Fig. 5.13) with length dz. Rays are directed to the observation point from an element in the beginning of the wire (point A) and from an element in an arbitrary point B on distance z from the beginning of a coordinate axis. The field intensity, which is created by the element in point B is

$$d\dot{E} = \frac{\dot{I}_z W\, dz}{2\lambda r_B}\sin\theta\, e^{-ikr_B}.$$

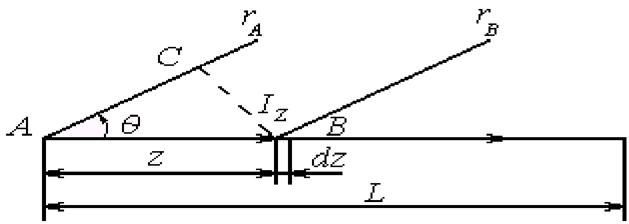


Fig. 5.13

The path-length difference from elements in points A and B can be defined from triangle ABC:

$$r_A - r_B = z\cos\theta.$$

Let us express the field intensity of element *B* through distance r_A in view of expressions (5.33) and (5.34):

$$d\dot{E} = i\frac{\dot{I}_A W\sin\theta}{2\lambda r}\, e^{-ikr_A}\, e^{-ikz(\xi - \cos\theta)}dz,$$

where r is the distance from an average point of the wire to the observation point.

The field intensity from all wire

$$\dot{E} = i\frac{\dot{I}_A W\sin\theta}{2\lambda r}\, e^{-ikr_A}\int_0^L e^{-ikz(\xi - \cos\theta)}dz,$$

where L is the length of the wire.

After integration we obtain

$$\dot{E} = E_{\max w} f(\theta)\, e^{i\psi(r,\theta)},$$

where the amplitude multiplier is

$$E_{\max w} = \frac{I_A L W}{2\lambda r},$$

DC is

$$f(\theta) = \sin\theta \frac{\sin\left[\frac{kL}{2}(\xi - \cos\theta)\right]}{\frac{kL}{2}(\xi - \cos\theta)} \qquad (5.35)$$

and the phase multiplier is

$$e^{i\psi(r,\theta)} = i e^{ikr_A} e^{i\frac{kL}{2}(\xi - \cos\theta)}.$$

The expression for DC of the wire with the running wave of current can be obtained directly with the help of the pattern multiplication theorem [formula (5.6)]. Really, the wire can be considered as an elementary radiator system with DC of kind $F(\theta) = \sin\theta$, which are continuously distributed along the straight line - axes of the wire - and fed by the current of the constant amplitude, but with the phase shift. In this case we should use formula (5.26) as the array factor. Multiplying DC of an electric dipole $(\sin\theta)$ by formula (5.26), we immediately come to expression (5.35).

Thus, DC of the wire with the running wave of current consists of two multipliers, one of which is the elementary radiator DC, the second – function $\sin u/u$, where $u = 0.5kL(\xi - \cos\theta)$. In the direction of axis $(\theta = 0)$ the wire radiation is absent as the first multiplier becomes equal to zero (an electric dipole does not radiate along an axis). At $\xi = 1$ the second multiplier accepts the maximal value at $\theta = 0$. In this connection the radiation maximums will be located under some angles to the wire axis. Approximately the direction of the major lobe maximum can be determined from the equation

$$\sin\left[\frac{kL}{2}(1 - \cos\theta_m)\right] = 1.$$

So, the argument of a sine should be equal to $\pi/2$, hence

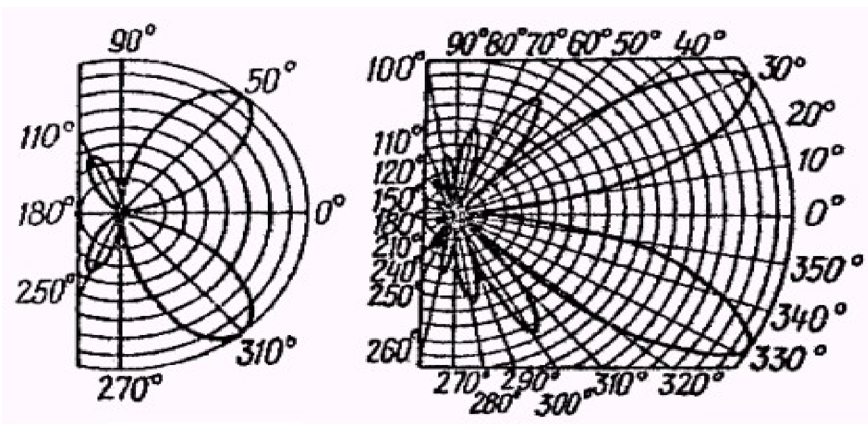
$$cos\,\theta_m = 1 - \frac{\lambda}{2L}.$$

In Fig. 5.*14(a)* the wire DC is presented at $\xi = 1$ and $L = \lambda$, in
Fig. 5.*14(b)* - at $\xi = 1$ and $L = 3\lambda$. DC indicates, that with lengthening wire the major lobes are narrowed and come nearer to the wire axis, the quantity of side lobes grows.

Fig. 5.14

5.7. Non-equal amplitude excitation array

Linear arrays with the uniform distribution are characterized by DC, which side lobes are of a significant level. Thus, as it follows from expressions (5.19), (5.21), the level of the first side lobe by approximate estimation



a *b*

$$\nu_1 = \frac{1}{n\,sin\left(\frac{3\pi}{2n}\right)}. \qquad (5.36)$$

With a considerable number of radiators n the sine argument will be very small and, consequently, it is possible to replace the sine by its argument. Then, from expression (5.36), the level of the first side lobe (the biggest) will take value $\nu_1 = 0.212$. More accurate calculation will give level of the first side lobe for an examined case: $\nu_1 = 0.217$ or $\nu_1 = -13.2\,\partial B$.

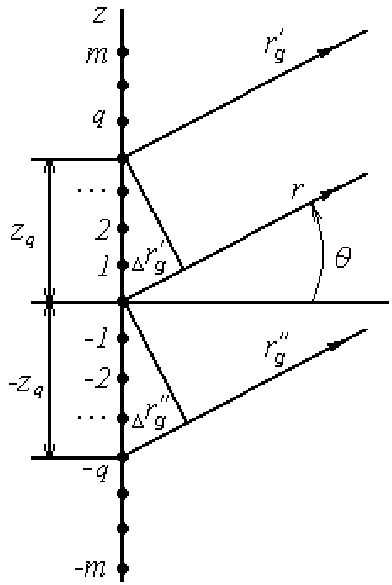


Fig. 5.15

Such level of side lobes for many radio engineering systems is unacceptable. Study of other types of the current distribution shows, that reduction of the side lobe level can be received by means of the feed current distribution with the non-equal amplitude, which falls down to the ends of the array.

Let us consider an array (Fig. 5.15) with the cosine-squared-on-a-pedestal distribution of the feed currents. Let the amount of radiators in an array will be even $n = 2m$, where m is an integer. A line of the radiator arrangement coincides with axis z and the origin of coordinate system is in the middle of the array. The radiators are numbered from the coordinate origin under the order from 1 to m aside both positive and negative values. The relative amplitude of the feed current of radiator q is

$$a_q = \delta + (1 - \delta)\cos^2\frac{\pi}{2}\left(\frac{2q-1}{n-1}\right), \qquad (5.37)$$

where δ is the pedestal height .

The current distribution in the array is presented in Fig. 5.16, where on axis z radiators coordinates are plotted, and on axis of ordinates values of relative current amplitudes a are plotted. As is seen from Fig. 5.16, for radiator q the value of coordinate is determined from the expression

$$z_q = (2q-1)\frac{d}{2}, \tag{5.38}$$

where d is the distance between radiators.

When the radiator is situated on the positive semi-axis Z, distance d is taken with the positive sign, in the opposite case – with negative. Thus, as it follows from section 5.1, the field distribution is defined by the array factor (5.5).

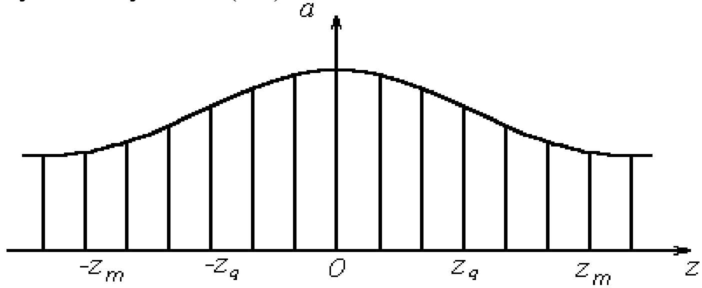


Fig. 5.16

Expression (5.37) is transformed with the aim to simplify the operation of summation

$$a_q = \frac{1+\delta}{2} + \frac{1-\delta}{2} cos\pi\left(\frac{2q-1}{n-1}\right).$$

For more compact record following designations are made:

$$t_1 = \frac{1+\delta}{2};\ \ t_2 = \frac{1-\delta}{2};\ \ \psi_0 = \frac{\delta}{n-1} \tag{5.39}$$

also, expressing the cosine function in the exponential form:

$$a_q = t_1 + \frac{t_2}{2} e^{i(2q-1)\varphi_0} + \frac{t_2}{2} e^{-i(2q-1)\varphi_0}. \tag{5.40}$$

Taking into account Fig. 5.15, we may write down, that

$$r_q' = r - z_q \sin\theta;\ \ r_q'' = r + z_q \sin\theta .$$

The path-length difference from the coordinate origin to the observation point and from radiator q to the same point is

$$\Delta r_q' = -\frac{d}{2}(2q-1)\sin\theta;$$
$$\Delta r_q'' = \frac{d}{2}(2q-1)\sin\theta. \tag{5.41}$$

Let us assume, that the phase of the feed current changes by the linear law

$$\psi_q = -\frac{2\pi}{\Lambda} z_q ,$$

where Λ is the wave length of the feed current at distribution along the array.

Entering wave slowing factor $\lambda/\Lambda = \xi = sin\,\theta_0$ and taking into account expression (5.54)

$$\varphi_q = -\frac{\pi d}{\lambda}(2q-1)\,sin\,\theta_0 \;\; at \;\; z_q > 0;$$
$$\varphi_q = \frac{\pi d}{\lambda}(2q-1)\,sin\,\theta_0 \;\; at \;\; z_q < 0. \qquad (5.42)$$

Angle θ_0 defines the direction of the aerial maximal radiation. An θ_0 is functionally connected with the phase shift of the feed current, therefore the direction of the maximal radiation in space can be changed by changing the phase shift. This circumstance is used for scanning in space.

Substituting expressions (5.40), (5.41) and (5.42) in formula (5.5)

$$f_\Sigma(u) = t_1\sum_{q=1}^{m} e^{i(2q-1)u} + \frac{t_2}{2}\sum_{q=1}^{m} e^{i(2q-1)(u+\varphi_0)} + \frac{t_2}{2}\sum_{q=1}^{m} e^{i(2q-1)(u-\varphi_0)} +$$
$$+ t_1\sum_{q=1}^{m} e^{-i(2q-1)u} + \frac{t_2}{2}\sum_{q=1}^{m} e^{-i(2q-1)(u+\varphi_0)} + \frac{t_2}{2}\sum_{q=1}^{m} e^{-i(2q-1)(u-\varphi_0)}, \qquad (5.43)$$

where u is the generalized angular argument:

$$u = \frac{\pi d}{\lambda}(sin\,\theta - sin\,\theta_0).$$

Any of these sums can be considered as a geometrical progression. Using the technique of summation, stated in section 5.3, we can bring equation (5.59) into the form:

$$f_\Sigma(u) = t_1 e^{i\frac{n}{2}u}\frac{sin\frac{nu}{2}}{sin\,u} + \frac{t_2}{2}e^{i\frac{n}{2}(u+\varphi_0)}\frac{sin\frac{n(u+\varphi_0)}{2}}{sin(u+\varphi_0)} +$$

$$+\frac{t_2}{2}e^{i\frac{n}{2}(u-\varphi_0)}\frac{\sin\frac{n(u-\varphi_0)}{2}}{\sin(u-\varphi_0)}+t_1e^{-i\frac{n}{2}u}\frac{\sin\frac{nu}{2}}{\sin u}+$$

$$+\frac{t_2}{2}e^{-i\frac{n}{2}(u+\varphi_0)}\frac{\sin\frac{n(u+\varphi_0)}{2}}{\sin(u+\varphi_0)}+\frac{t_2}{2}e^{-i\frac{n}{2}(u-\varphi_0)}\frac{\sin\frac{n(u-\varphi_0)}{2}}{\sin(u-\varphi_0)}.$$

Summarizing members at identical factors t_i and replacing these factors by their values from formulas (5.55) we obtain

$$f_\Sigma(u)=\frac{1+\delta}{2}\frac{\sin nu}{\sin u}+\frac{1-\delta}{4}\left[\frac{\sin n(u+\varphi_0)}{\sin(u+\varphi_0)}+\frac{\sin n(u-\varphi_0)}{\sin(u-\varphi_0)}\right]. \quad (5.44)$$

From expression (5.60) we can pass to the formula, which describes the array factor at continuous distribution of field sources. At such transition $n\to\infty$ and $d\to 0$. Let us designate nu through ϑ, then, as it follows from the formula for the generalized angular argument

$$nu=\vartheta=\frac{\pi L}{\lambda}(\sin\theta-\sin\theta_0),$$

where $L=nd$ is the aerial length.

Then the array factor

$$f_\Sigma(\nu)=(1+\delta)\frac{\sin\vartheta}{\vartheta}-(1-\delta)\frac{\vartheta\sin\vartheta}{\vartheta^2-\pi^2}. \quad (5.45)$$

Analyzing expression (5.45) we can find values of the beamwidth and a level of the first side lobe as a function of pedestal δ (Tab. 5.1).

Table 5.1

Pedestal value	0	0,2	0,4	0,6	0,8	1,0
Beamwidth $2\theta_{0.5}$	$83°\frac{\lambda}{L}$	$67.5°\frac{\lambda}{L}$	$60.5°\frac{\lambda}{L}$	$56°\frac{\lambda}{L}$	$52.5°\frac{\lambda}{L}$	$50.5°\frac{\lambda}{L}$
Side lobe level, ν_1, dB	-32	-30,3	-24,3	-18,7	-15,2	-13,2

It is possible to conceptually solve a problem of search of such current distribution in the aerial array, at which the level of side lobes will be zero. It is a special case of Dolf-Chebyshev distribution.

Let the distance between radiators satisfy to condition $d \leq \lambda/2$. For the cophased array we may take $d = \lambda/2$. Let us consider an opportunity of creation of distribution, at which side lobes will be absent. Such distribution is referred to as binomial, and the ratio of current amplitudes in array elements is determined by binomial factors (Tab. 5.2).

Table 5.2.

Number of radiators	Relative amplitude distribution
1	1
2	1 1
3	1 2 1
4	1 3 3 1
5	1 4 6 4 1
6	1 5 10 10 5 1
7	1 6 15 20 15 6 1

The array system can be found rather simply. From equation (5.18) at $n = 2$

$$F_{\Sigma}(u) = cos\frac{u}{2}, \qquad (5.46)$$

where $u = kd\,cos\,\theta - \psi$.

At $n = 3$ the array can be considered as two equal amplitude excitation arrays consisting of two radiators and located on one straight line so that the last adjacent radiators of both arrays have coincided with each other. Then the current distribution will correspond to the third line of tab. 5.2. The distance between centres of arrays is d. Therefore according to the multiplication theorem it is possible to find DC as product of a single element array DC (5.46) by a multiplier of two-radiator system.

So, for three radiators with the binomial current distribution

$$F_{\Sigma}(u) = cos^2\frac{u}{2}.$$

At four radiators $(n = 4)$ with the binomial current distribution the array can be considered as a set of two arrays, each consisting of three radiators with the binomial current distribution, which centres are located on distances d . So, DC will be described by the formula

$$F_{\Sigma}(u) = cos^{3}\frac{u}{2}.$$

Similarly, expressions for arrays with a greater number of radiators can be gained. Generally a multiplier of system from n radiators

$$F_{\Sigma}(u) = cos^{n-1}\frac{u}{2}. \qquad (5.47)$$

If to choose distance correctly, the multiplier (5.46) will not have side lobes and DC of array with the binomial current distribution (5.47) will be free from side lobes.

In practice the binomial distribution finds the limited application; the reason is rather wide DC and difficulty of realization of the current distribution. The last remark is of special value at a big number of radiators. For example, at $n = 7$ (Tab. 5.2) it is necessary to establish the ratio of current amplitudes one to twenty. Therefore it is more expedient to choose other kinds of distribution.

5.8. Nonuniformly spaced array

The low level of side lobes can be received with the equal amplitude excitation arrays by placing radiators on unequal distances. Such arrays are called nonuniformly spaced. Pattern of nonuniformly spaced arrays at given beamwidth and side lobes level is formed at a smaller number of radiators, than in equal amplitude excitation arrays. Violation of periodicity in a radiation arrangement results in suppression of the diffraction major lobes of the higher orders. This property of the nonuniformly spaced array is very useful for scanning of a beam in wide limits.

The radiation field of non-uniformly spaced arrays can be determined generally by formula (5.4). At a large quantity of elements such problem is solved very hardly and, as a rule, by numerical methods.

For designing of nonuniformly spaced arrays there have been developed methods in which such techniques are used:

- Comparison of the density of the radiators' arrangement with the amplitude distribution of the current following lengthwise of uniformly spaced arrays, which characteristics are to be received in the designed aerial;

- Variations of the radiators' position in an array;
- Series approximation of the array factor;
- Statistical calculations, etc.

One of such methods (rather simple) is the method of the current moments. According to this method the necessary amplitude distribution in uniformly spaced arrays is found from given DD (Fig. 5.*17(a)*). On axis Z coordinates z_S $(s = 0,1,2\ldots)$ specify positions of the array elements.

Distances between them are identical and equal to d. The relative amplitude of a feed current of each element is designated by a piece of a vertical straight line, which length is proportional to the current $I_S = f(z_S)$.

On preset values of current $f(z_S)$ we build a continuous function $f(z)$ and found the area $S_0 = \int_0^{z_n} f(z)dz$, which is limited by curve $f(z)$ at $0 \le z \le z_n$ (Fig. 5.*17(b)*). Choosing an average distance

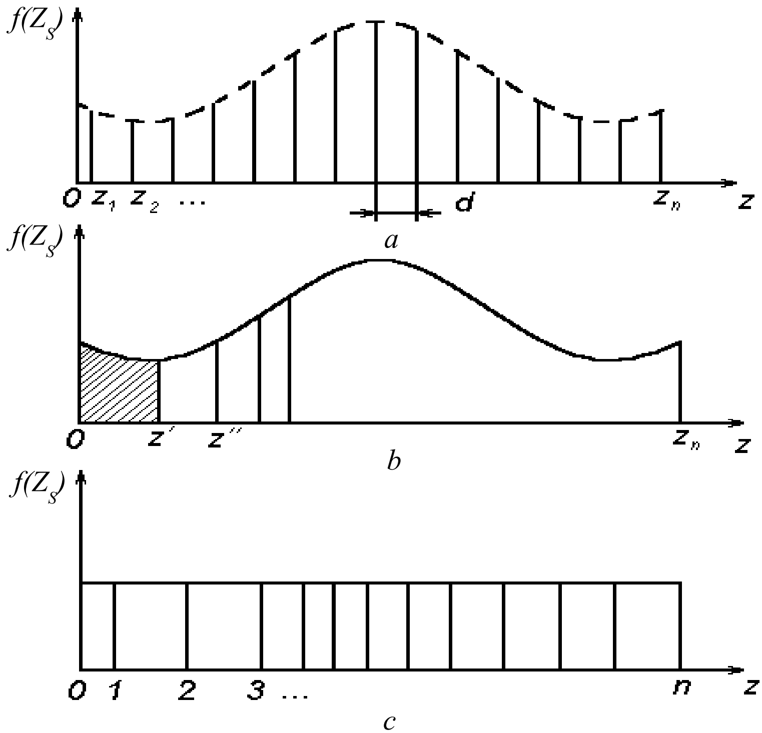


Fig. 5.17

between radiators of nonuniformly spaced arrays from 0,5 up to 1,0 λ, we can determine the number of radiators from the ratio

$$N = \frac{nd}{d_{av}}.$$

Let us divide area S_0 into N equal parts:

$$S_1 = \frac{S_0}{N} = \frac{1}{N}\int_0^{z_n} f(z)dz$$
.

If $F(z) = \int_0^{z_n} f(z)dz$, then positions of points $z', z'', \ldots$, which limit sections of the equal areas, can be found from expressions

$S_1 = F(z') - F(0)$ - for the first section;

$S_1 = F(z'') - F(z')$ - for the second section, etc.

Since in general sections are not rectangular, let us find the centre of gravity for each of sites. Coordinates of the centres of gravity will be also radiators coordinates in nonuniformly spaced array (Fig. 5.*17(c)*). It is known, that the field distribution in space is proportional to the integral from distribution of current amplitudes, therefore as a result of such calculation we receive the nonuniformly spaced array with the uniform current distribution and the radiation characteristics, equivalent to the characteristics of the nonequal amplitude excitation and equally spaced array.

In the statistical method a stochastic function of radiators arrangement is chosen. It is considered that radiators are fed by cophased currents of equal amplitudes. By using of probabilistic methods the distribution of the DC random function is defined and its parameters are estimated.

The beamwidth of the nonuniformly spaced array basically depends on the ratio of its length to the wavelength and weakly depends on the character of the radiators arrangement. In this connection at given DC the quantity of elements in the nonuniformly spaced array can be smaller than in the uniformly spaced array. Besides, nonuniformly spaced arrays work in a wider frequency range than uniformly spaced arrays.

5.9. Two-radiator system

Aerial devices, which consist of two radiators, are rather widely used in radio engineering. By their principle of operation a number of antennas can be treated as a two-radiator system. Taking into account practical significance of such system, let us consider it in detail.

At $n=2$ the array factor (5.16) takes the form

$$F_\Sigma(u)=\frac{sin\left[\frac{2}{2}(kd\,cos\,\theta-\psi)\right]}{2\,sin\left[\frac{1}{2}(kd\,cos\,\theta-\psi)\right]}=cos\left[\frac{1}{2}(kd\,cos\,\theta-\psi)\right].$$

Directions of zero are determined from formula (5.23), which in view of restriction on value p can be written down as

$$cos\,\theta_{0p}=\frac{p\pi}{kd}+\frac{\psi}{kd};\ p=\pm1,\pm3,\pm5,\dots.$$

For a cophased system at $\psi=0$

$$F_\Sigma(\theta)=cos\left(\frac{kd\,cos\,\theta}{2}\right). \qquad (5.48)$$

In this case one of the maximal radiation directions is guided perpendicularly to the axis of system:

$$cos\,\theta_{max,p}=\frac{p\lambda}{d};\ where\ p=0,\pm1,\pm2,\dots.$$

In Fig. 5.18*(a)* DDs of the cophased system in a plane, perpendicular to radiators are represented, and in Fig. 5.*18(b)* - in a plane, including axes of parallel dipoles. From Fig. 5.18 it is obvious, that the distance between radiators essentially influences the DD form. Directions of zero in DD are determined from the expression

$$cos\,\theta_{0p}=(2p+1)\frac{\lambda}{2d};\ at\ p=0,\pm1,\pm2,\dots.$$

At $\psi=\pi/2$ and $d=\lambda/4$ the axial radiation array is obtained

$$F_\Sigma(\theta)=cos\left[\frac{\pi}{4}(1-cos\,\theta)\right].$$

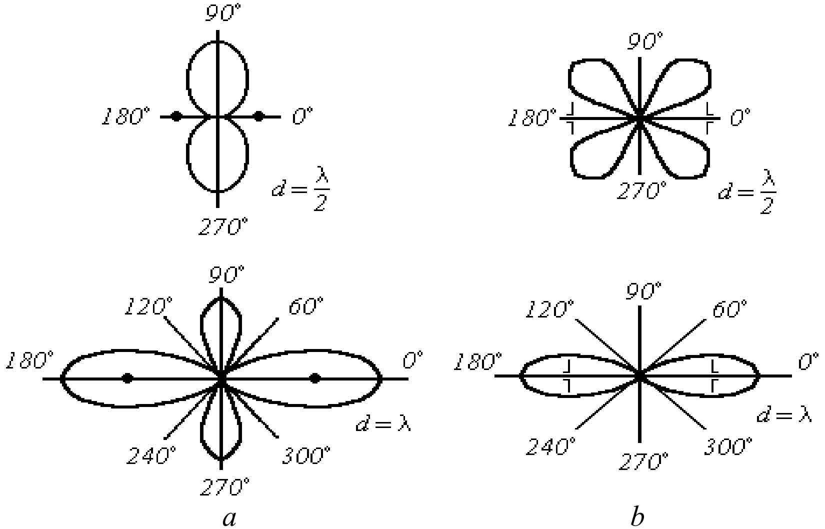


Fig. 5.18

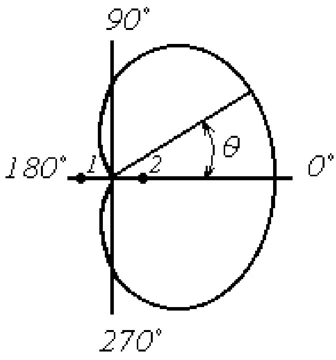


Fig. 5.19

The directional diagram of such aerial is presented in Fig. 5.19. Radiator 1 is reflector in relation with radiator 2. Thus, the current in the reflector should lead the current in the radiator 2 by $90°$.

For an antiphase system at $\psi = \pi$

$$F_{\Sigma}(\theta) = sin\left(\frac{kd\cos\theta}{2}\right).$$

The maximum radiation direction is determined through distance between dipoles

$$cos\,\theta_{max,p} = (2p+1)\frac{\lambda}{2d};\ at\ \ p = 0, \pm 1, \pm 2, \ldots .$$

(5.49)

Directions of the zero radiation also depend on distance d :

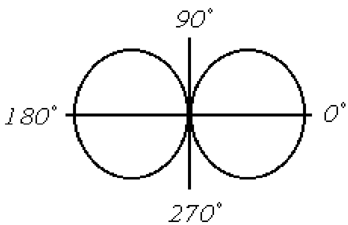
$$\cos\theta_{0p} = \frac{p\lambda}{d};\ where\ p = 0, \pm 1, \pm 2, \ldots . \qquad (5.50)$$

Concerning the cophased system (5.48) in the antiphase system the DC values of maximum and zero have exchanged their places. Therefore, as it follows from formula (5.50), there will always be zero of radiation in the direction of the perpendicular to the axis of system.

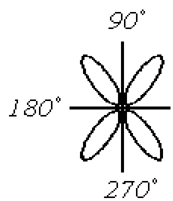
In Fig. 5.20 the antiphase system DDs for two values of distance between radiators are presented: *a* - for $d = \lambda/2$ and *b* - for $d = \lambda$.

In Fig. 5.21 the system of two radiators, fed by different currents $\dot{I}_1$ and $\dot{I}_2$ is shown. Let us assume, that DDs of both radiators are identical and described by function $F_1(\theta,\varphi)$. The ratio of currents can be expressed as:

$$\frac{\dot{I}_2}{\dot{I}_1} = me^{-i\psi}. \qquad (5.51)$$



a



b

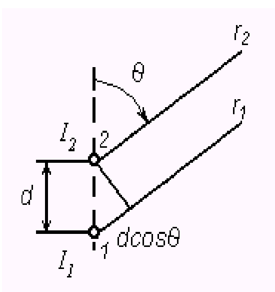


Fig. 5.20

Fig. 5.21

The field intensity in the observation point is determined by the field intensity of the first and the second radiators

$$\dot{E}_1 = i\frac{60\dot{I}_1}{r_1}F_1(\theta,\varphi)e^{-ikr_1};$$
$$\dot{E}_2 = i\frac{60\dot{I}_2}{r_2}F_1(\theta,\varphi)e^{-ikr_2}.$$

Taking into account the path-length difference, let us obtain the sum of intensity:

$$\dot{E} = \dot{E}_1 + \dot{E}_2 = i\frac{60}{r}\dot{I}_1F_1(\theta,\varphi)e^{-ikr}\left[1 + me^{i(kd\cos\theta - \psi)}\right], \quad (5.52)$$

where r is the distance from an average point of the system to the observation point.

From expression (5.52) the array factor can be determined

$$f(\theta) = \left|1 + me^{i(kd\cos\theta - \psi)}\right| = \sqrt{1 + m^2 + 2m\cos(kd\cos\theta - \psi)}. \quad (5.53)$$

Formula (5.53) is used for calculation of the directional properties of the two-element system with a passive reflector or a passive director. In this case the current is fed only to one of dipoles. The ratio of radiator currents is found from the set of equations (5.7), which, for the given case, when the EMF source feeds only the first radiator, takes the following form:

$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2;$$
$$0 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2.$$

Hence it follows, that

$$\dot{I}_2 = -\frac{Z_{21}}{Z_{22}}\dot{I}_1. \quad (5.54)$$

Comparing expressions (5.51) and (5.54) and taking into account, that $Z_{12} = Z_{21}$

$$m = \sqrt{\frac{R_{12}^2 + X_{12}^2}{R_{22}^2 + X_{22}^2}};$$
$$\psi = \pi + arctg\left(\frac{X_{12}}{R_{12}}\right) - arctg\left(\frac{X_{22}}{R_{22}}\right). \quad (5.55)$$

The ratio of modules of currents m and the phase shift between them depend on the distance between dipoles d and the arms' length of the second dipole. At some values m and Ψ such system will have the directed properties, very close to the properties of the equal amplitude excitation array with the axial radiation.